Title: Making Waves

Brief Overview:

The students will construct and analyze equations of trigonometric functions from given tables of temperature data, observe patterns, and use regression analysis on the TI-83 calculator. They will explore phase shifts and changes in amplitude by analyzing the graphs of the normal daily maximum temperature average of U.S. cities from 1961-1990. Through the assessment they will make predictions using the regression equation.

NCTM 2000 Principles for School Mathematics:

- **Equity:** *Excellence in mathematics education requires equity - high expectations and strong support for all students.*
- **Curriculum:** *A curriculum is more than a collection of activities: it must be coherent, focused on important mathematics, and well articulated across the grades.*
- **Teaching:** *Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well.*
- **Learning:** *Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge.*
- **Assessment:** *Assessment should support the learning of important mathematics and furnish useful information to both teachers and students.*
- **Technology:** *Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning.*

Links to NCTM 2000 Standards:

- **Content Standards**

Algebra

Students will understand patterns, relations, and functions and use mathematical models to represent and understand quantitative relationships.

Data Analysis and Probability

Students will use scatter plots to display data, identify trends, and find functions that model the data.

- **Process Standards**

  <u>Mathematics as Problem Solving, Reasoning and Proof, Communication, Connections, and Representation</u>
  These five process standards are threads that integrate throughout the unit, although they may not be specifically addressed in the unit. They emphasize the need to help students develop the processes that are the major means for doing mathematics, thinking about mathematics, understanding mathematics, and communicating mathematics.

  Students will be able to develop patterns among and complete regression analysis on data. They will be able to explain the relationship between data points and regression equations. They also will be required to write about patterns generated from data and to justify their conclusions. Lastly, they will demonstrate knowledge of representing data in multiple forms including equations and graphs.

Links to Maryland High School Mathematics Core Learning Units:

Functions and Algebra

- **1.1.1**
  Students will recognize, describe, and extend patterns and functional relationships that are expressed numerically, algebraically, and geometrically.

- **1.1.2**
  Students will represent patterns and functional relationships in a table, as a graph, and/or by mathematical expression.

Data Analysis and Probability

- **3.1.1**
  Students will design and/or conduct an investigation that uses statistical methods to analyze data and communicate results.

Links to Virginia High School Mathematics Standard of Learning:

- **T.6, AII/T.26**
  Students, given one of the six trigonometric functions in standard form (e.g., $y = A\sin(Bx+C) + D$, where A, B, C, and D are real numbers), will determine the amplitude, period, phase shift, and vertical shift.

Grade/Level:

Grades 11-12; Trigonometry/Pre-Calculus

Duration/Length:

Two 90 minutes block periods

Prerequisite Knowledge:

Students should have working knowledge of the following skills:

- Inputting data into the TI-83 and creating a scatter plot
- Recognizing and identifying graphs of trigonometric functions, e.g., the sine function
- Performing a regression analysis on the TI-83
- Identifying the period, amplitude, and phase shift of trigonometric functions

Student Outcomes:

Students will be able to:

- interpret data they are given to draw conclusions.
- determine the average from a set of data they are given.
- use the graphing calculator to input lists, create scatter plots, and calculate regression lines.
- discuss phase shift and amplitude when comparing graphs of different data.
- predict outcomes using the regression equation.

Materials/Resources/Printed Materials:

- Warm-up
- Student Activity Worksheets A & B, Homework Sheet
- Procedures for creating scatter plots and calculating regression lines on TI-83
- TI-83 graphing calculator

Development/Procedures:

The teacher should lead the class in a discussion on weather and weather patterns, such as asking questions on the following: how does the average monthly temperature in our area change over the course of a year? and what would you predict the graph of average monthly temperature vs. time would to look like? The teacher should then review the sine function. The warm-up provides the student with practice in finding the amplitude, period, and phase shift. The unit consists of parallel Student Activity Worksheets A and B. The teacher can choose to require the students to complete both worksheets or divide the class into groups of four students. One pair in the group may work on Worksheet A and the other on Worksheet B. At the end of the activity, the pairs should share their findings with the class. The homework serves to strengthen the skills developed through the activity.

Assessment:

The students independently will implement the skills learned in the lesson by analyzing data collected on the waxing and waning of tides. The assessment will take approximately 30 minutes.

Extension/Follow Up:

- Students can present their findings from the activities by constructing a poster or PowerPoint presentation.
- Students can use the Internet to research a different periodic event, such as daily sunrise times, and perform a similar analysis.
- Students can explore sound waves and swinging of a pendulum using data collection devices.

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Warm Up

1. The period of $y = 2\cos(2x)$ is:

2. Find the period of $y = -\sin(x - p)$.

3. The amplitude of $y = 52.4\sin(1.2x + 3p)$ is:

4. The amplitude of $y = -0.543\cos(0.5x) + 15$ is:

5. The maximum value of $y = -0.543\cos(0.5x) + 15$ is:

Warm Up
Answer Key

1. The period of $y = 2\cos(2x)$ is: **π .**

2. Find the period of $y = -\sin(x - p)$. **2π .**

3. The amplitude of $y = 52.4\sin(1.2x + 3p)$ is: **52.4 .**

4. The amplitude of $y = -0.543\cos(0.5x) + 15$ is: **0.543 .**

5. The maximum value of $y = -0.543\cos(0.5x) + 15$ is: **15.543 .**

PROCEDURE FOR PRODUCING SCATTER PLOT:

a. Press [STAT], [ENTER].
b. Input number of month in L1.
c. Input normal daily maximum temperature for the first city in L2.
d. Input normal daily maximum temperature for the second city in L3.
e. Press [2^{nd}], [STAT PLOT], [1], [ENTER].
f. Turn scatter plot ON.
g. Select scatter plot icon.
h. Choose L1 and L2.
i. Choose the first symbol.
j. Press [2^{nd}], [STAT PLOT], [2], [ENTER].
k. Turn scatter plot ON.
l. Select scatter plot icon.
m. Choose L1 and L3.
n. Choose the second symbol.
o. Press [ZOOM], [9].

PROCEDURE FOR CONDUCTING REGRESSION:

a. List 2 vs. List 1
From home screen, press [STAT], [CALC], [ALPHA], [C], [2^{nd}], [1], [,], [2^{nd}], [2], [,], [VARS], [▸], [ENTER], [ENTER], [ENTER].

b. List 3 vs. List 1
From home screen, press [STAT], [CALC], [ALPHA], [C], [2^{nd}], [1], [,], [2^{nd}], [3], [,], [VARS], [▸], [ENTER], [ENTER], [ENTER].

c. Press [ZOOM], [9].

Name: _____ Date: _____

STUDENT ACTIVITY SHEET A

The following chart displays the 1961-1990 Normal Daily Maximum Temperature in degrees Fahrenheit for Barrow, Alaska and Honolulu, Hawaii. The information was obtained from the National Weather Service at http://www.nws.mbay.net/maxtemp.html.

City	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Barrow,AK	-7.4	-11.8	-9	4.7	24.2	38.3	45	42.3	33.8	18.1	3.5	-5.2
Honolulu,HI	80.1	80.5	81.6	82.8	84.7	86.5	87.5	88.7	88.5	86.9	84.1	81.2

1. Looking at the temperature data for Alaska, what do you predict that the graph of the data will look like?

2. Looking at the temperature data for Hawaii, what do you predict that the graph of the data will look like?

3. Calculate the average of the monthly temperatures given for each city. Explain the method you used to calculate this average.

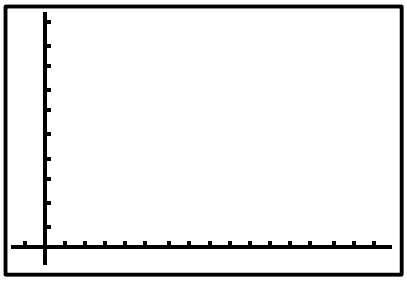
   Barrow, AK _____

   Honolulu, HI _____

4. Find the highest and lowest temperature for each city. Calculate their distance from the average temperature calculated in Question 3.

		Distance From Average | temp. – avg. |
Barrow, AK	high_____	_____
	low _____	_____
Honolulu, HI	high_____	_____
	low _____	_____

5. A. On the same axes produce two scatter plots from the data given in the chart. Sketch your results below. Be sure to label the axes clearly.
Do the graphs match your predictions? Explain.



B. What type of regression do you think will best fit the data?_____

6. Perform this regression on each data set. Make sure your calculator is in radian mode.

A. What does x represent? _____

B. What does y represent? _____

C. Record your regression equations below. Round all decimals to the nearest hundredth.

Barrow, AK _____

Honolulu, HI _____

D. Sketch the regression curve with the scatter plot in Question 5A.

7. A. Using the regression equations, what is the amplitude of each function?

Barrow, AK _____

Honolulu, HI _____

B. Explain what the amplitude means in terms of deviation from the average temperature. [Hint: See Question 4]

8. A. Using the regression equations, what is the period of each function?

Barrow, AK _____

Honolulu, HI _____

B. Why do these values make sense in terms of our temperature data?

9. Look at the constant term (d) in each equation.

   A. How does the constant term translate the graph?

   B. Explain what the constant term means in terms of temperature.
   [Hint: See Question 3]

10. A. In what month(s) do the maximum and minimum temperatures occur?

   Maximum _____ Minimum _____

   B. Would you expect this to be the same for any city in the U.S.? Explain.

   C. Would you expect this to be the same for any city in the world? Explain.

11. Write a paragraph comparing the climates of Barrow, AK and Honolulu, HI using the information found in this exploration. Include both similarities and differences.

STUDENT ACTIVITY SHEET A - KEY

The following chart displays the 1961-1990 Normal Daily Maximum Temperature in degrees Fahrenheit for Barrow, Alaska and Honolulu, Hawaii. The information was obtained from the National Weather Service at http://www.nws.mbay.net/maxtemp.html.

City	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Barrow,AK	-7.4	-11.8	-9	4.7	24.2	38.3	45	42.3	33.8	18.1	3.5	-5.2
Honolulu,HI	80.1	80.5	81.6	82.8	84.7	86.5	87.5	88.7	88.5	86.9	84.1	81.2

1. Looking at the temperature data for Hawaii, what do you predict that the graph of the data will look like? **See student answer.**

2. Looking at the temperature data for Alaska, what do you predict that the graph of the data will look like? **See student answer.**

3. Calculate the average of the monthly temperatures given for each city.

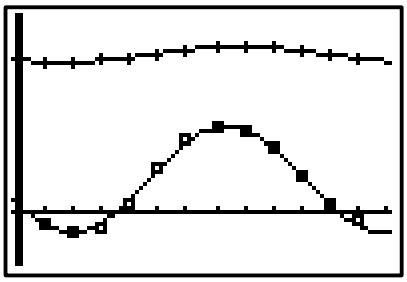
   Barrow, AK __**14.7**___ **Students may use 1-Var Stats or add all of the numbers and divide by 12.**

   Honolulu, HI __**84.4**___

4. Find the highest and lowest temperature for each city. Calculate their distance from the average temperature calculated in Question 3.

		Distance From Average | temp. – avg. |
Barrow, AK	high__**45.0**____	__**30.3**____
	low __**-11.8**____	__**26.5**____
Honolulu, HI	high__**88.7**____	___**4.3**____
	low __**80.1**____	___**4.3**____

5. A. Produce two scatter plots from the data given in the chart. Sketch your results below. Do the graphs match your predictions? Explain.



See student answer.

X is number of month.
Y is avg. temp. in degrees Fahrenheit.
Xmin=-.1 Xmax=13.1
Ymin=-28.9 Ymax=105.8

B. What type of regression do you think will best fit the data? **sinusoidal**

6. Perform this regression on each data set. Make sure your calculator is in radian mode.

   A. What does x represent? **number of month (1=Jan, 2=Feb, etc.)**

   B. What does y represent? **normal daily max temp. in degrees Fahrenheit**

   C. Record your regression equations below. Round all decimals to the nearest hundredth.
   Barrow, AK **y = 28.53 sin(.58x – 2.70) + 17.00**

   Honolulu, HI **y = 4.24 sin(.51x – 2.44) + 84.33**

   D. Sketch the regression curve with the scatter plot in Question 5A.
   See question 5A.

7. A. Using the regression equations, what is the amplitude of each function?

   Barrow, AK **28.53**

   Honolulu, HI **4.24**

   B. Explain what the amplitude means in terms of deviation from the average temperature. [Hint: See Question 4]
   The amplitude represents the degrees the temperature varied above and below the average temperature.

8. A. Using the regression equations, what is the period of each function?

   Barrow, AK **$2\pi/.58 = 10.83$ months**

   Honolulu, HI **$2\pi/.51 = 12.32$ months**

   B. Why do these values make sense in terms of our temperature data?
   Since the period is approximately 12 months, the data shows that one complete temperature cycle takes about 1 year.

9. Look at the constant term in each equation. **AK = 17.00, HI = 84.33**

   A. How does the constant term translate the graph? **The constant term shifts the graph up or down.**
   B. Explain what the constant term means in terms of temperature.
   [Hint: See Question 3]
   The constant term/upward shift corresponds with the average temperature for the year.

10. A. In what month(s) do the maximum and minimum temperatures occur?

    Maximum _**July, August**___ Minimum _**February, January**__

    B. Would you expect this to be the same for any city in the U.S.? Explain.
    Yes. Most or all U.S. cities have their hottest temperatures in the summer and their coolest temperatures in the winter.

    C. Would you expect this to be the same for any city in the world? Explain.
    No. Cities in the southern hemisphere have opposite warm and cool seasons.

11. Write a paragraph comparing the climates of Barrow, AK and Honolulu, HI using the information found in this exploration. Include both similarities and differences.
    Answers should include the following:
    Similarities – periodic, high temps in summer, low temps in winter
    Differences – AK has a much lower average temperature than HI. AK's temperatures vary a lot from the average, while HI has only slight variations from the average.

Name: _____ Date: _____

STUDENT ACTIVITY SHEET B

The following chart displays the 1961-1990 Normal Daily Maximum Temperature in degrees Fahrenheit for Topeka, Kansas and San Francisco, California. The information was obtained from the National Weather Service at http://www.nws.mbay.net/maxtemp.html.

City	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Topeka, KS	37	42.6	55	66.9	75.8	84.2	89.3	87.5	79.7	69	54	40.5
San Fran.,CA	55.6	59.4	60.8	63.9	66.5	70.3	71.6	72.3	73.6	70.1	62.4	56.1

1. Looking at the temperature data for Topeka, what do you predict that the graph of the data will look like?

2. Looking at the temperature data for San Francisco, what do you predict that the graph of the data will look like?

3. Calculate the average of the monthly temperatures given for each city.

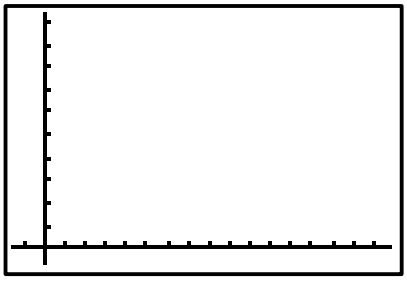
   Topeka, KS _____

   San Francisco, CA _____

4. Find the highest and lowest temperature for each city. Calculate their distance from the average temperature calculated in Question 3.

		Distance From Average | temp. – avg. |
Topeka, KS	high_____	_____
	low _____	_____
San Fran., CA	high_____	_____
	low _____	_____

5. A. On the same axes produce two scatter plots from the data given in the chart.
   Sketch your results below. Be sure to label the axes clearly.
   Do the graphs match your predictions? Explain.



B. What type of regression do you think will best fit the data? _____

6. Perform this regression on each data set. Make sure your calculator is in radian mode.

   A. What does x represent? _____

   B. What does y represent? _____

   C. Record your regression equations below. Round all decimals to the nearest hundredth.

      Topeka, KS _____

      San Fran., CA _____

   D. Sketch the regression curve with the scatter plot in Question 5A.

7. A. Using the regression equations, what is the amplitude of each function?

      Topeka, KS _____

      San Fran., CA _____

   B. Explain what the amplitude means in terms of deviation from the average temperature. [Hint: See Question 4]

8. A. Using the regression equations, what is the period of each function?

      Topeka, KS _____

      San Fran., CA _____

   B. Why do these values make sense in terms of our temperature data?

9. Look at the constant term (*d*) in each equation.

   A. How does the constant term translate the graph?

   B. Explain what the constant term means in terms of temperature.
      [Hint: See Question 3]

10. A. In what month(s) do the maximum and minimum temperatures occur?

Maximum _____ Minimum _____

B. Would you expect this to be the same for any city in the U.S.? Explain.

C. Would you expect this to be the same for any city in the world? Explain.

11. Write a paragraph comparing the climates of Topeka, KS and San Fran. , CA using the information found in this exploration. Include both similarities and differences.

STUDENT ACTIVITY SHEET B - KEY

The following chart displays the 1961-1990 Normal Daily Maximum Temperature in degrees Fahrenheit for Topeka, Kansas and San Francisco, CA. The information was obtained from the National Weather Service at http://www.nws.mbay.net/maxtemp.html.

City	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Topeka, KS	37	42.6	55	66.9	75.8	84.2	89.3	87.5	79.7	69	54	40.5
San Fran.,CA	55.6	59.4	60.8	63.9	66.5	70.3	71.6	72.3	73.6	70.1	62.4	56.1

1. Looking at the temperature data for Topeka, what do you predict that the graph of the data will look like? **See student answer.**

2. Looking at the temperature data for San Francisco, what do you predict that the graph of the data will look like? **See student answer.**

3. Calculate the average of the monthly temperatures given for each city.

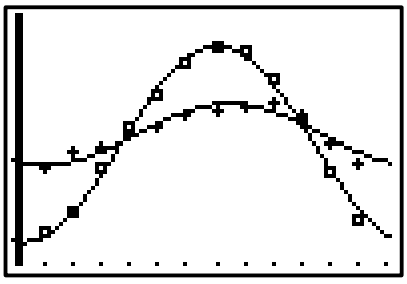
   Topeka, KS __**65.1**___ **Students may use 1-Var Stats or add all of the numbers and divide by 12.**

   San Fran., CA __**65.2**___

4. Find the highest and lowest temperature for each city. Calculate their distance from the average temperature calculated in Question 3.

		Distance From Average | temp. – avg. |
Topeka, KS	high__**89.3**____	__**24.2**____
	low __**37.0**____	__**28.1**____
San Fran., CA	high__**73.6**____	___**8.4**___
	low __**55.6**____	___**9.6**____

5. A. Produce two scatter plots from the data given in the chart. Sketch your results below. Do the graphs match your predictions? Explain.



See student answer.

X is number of month.
Y is avg. temp. in degrees Fahrenheit.
Xmin=-.1 Xmax=13.1
Ymin=28.1 Ymax=98.2

B. What type of regression do you think will best fit the data? **sinusoidal**

6. Perform this regression on each data set. Make sure your calculator is in radian mode.

   A. What does x represent? **number of month (1=Jan, 2=Feb, etc.)**

   B. What does y represent? **normal daily max temp. in degrees Fahrenheit**

   C. Record your regression equations below. Round all decimals to the nearest hundredth.
   Topeka, KS **y = 27.48 sin(.46x – 1.69) + 61.70**

   San Fran., CA **y = 8.51 sin(.49x – 2.08) + 64.67**

   D. Sketch the regression curve with the scatter plot in Question 5A.
   See question 5A.

7. A. Using the regression equations, what is the amplitude of each function?

   Topeka, KS **27.48**

   San Fran., CA **8.51**

   B. Explain what the amplitude means in terms of deviation from the average temperature. [Hint: See Question 4]
   The amplitude represents the degrees the temperature varied above and below the average temperature.

8. A. Using the regression equations, what is the period of each function?

   Topeka, KS **$2\pi/.46 = 13.66$ months**

   San Fran., CA **$2\pi/.49 = 12.82$ months**

   B. Why do these values make sense in terms of our temperature data?
   Since the period is approximately 12 months, the data shows that one complete temperature cycle takes about 1 year.

9. Look at the constant term in each equation. **KS = 61.70, CA = 64.67**

   A. How does the constant term translate the graph? **The constant term shifts the graph up or down.**
   B. Explain what the constant term means in terms of temperature.
   [Hint: See Question 3]
   The constant term/upward shift corresponds with the average temperature for the year.

10. A. In what month(s) do the maximum and minimum temperatures occur?

    Maximum **July, August** Minimum **January**

    B. Would you expect this to be the same for any city in the U.S.? Explain.
    Yes. Most or all U.S. cities have their hottest temperatures in the summer and their coolest temperatures in the winter.

    C. Would you expect this to be the same for any city in the world? Explain.
    No. Cities in the southern hemisphere have opposite warm and cool seasons.

11. Write a paragraph comparing the climates of Topeka, KS and San Fran., CA using the information found in this exploration. Include both similarities and differences.
    Answers should include the following:
    Similarities – periodic, high temps in summer; low temps in winter; and average temperature for the year is almost exactly the same.
    Differences – Topeka's temperatures vary widely from the average temperature, San Francisco's temperatures do not vary as much from the average.

Homework Assignment

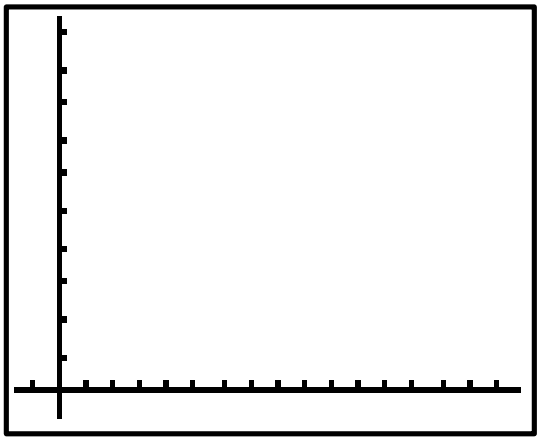
Phase: phase is the shift of a sine wave expressed in terms of radians or degrees.

Amplitude: The amplitude of a function is one half the difference between the maximum and the minimum value. The amplitude of a periodic function with maximum value A and minimum value a is

$$\frac{A - a}{2}$$

City	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Topeka, KS	37	42.6	55	66.9	75.8	84.2	89.3	87.5	79.7	69	54	40.5
Barrow, AK	-7.4	-11.8	-9	4.7	24.2	38.3	45	42.3	33.8	18.1	3.5	-5.2

1. Using the graphing calculator create a scatter plot and regression line of temperatures for Topeka, KS and Barrow, AK. Use the graph below to reproduce what you see on the calculator screen. Be sure to label your axes with variables and scales.



2. Calculate the amplitude using the formula above for each set of data. Compare your answers with the amplitudes from the equation you discovered using the calculator in problem number 1.

3. Compare and discuss the phase shift and amplitude of the two graphs. The discussion should include the similarities and differences of the two graphs.

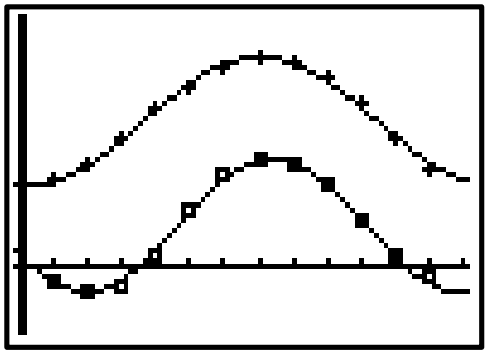
4. If the following equation represents the pattern of temperature changes in a certain city in the United States, what would the average daily temperature be in June?

$$y = 23.58\sin(.55x - 2.33) + 81.49$$

Homework Assignment: Key

City	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Topeka, KS	37	42.6	55	66.9	75.8	84.2	89.3	87.5	79.7	69	54	40.5
Barrow, AK	-7.4	-11.8	-9	4.7	24.2	38.3	45	42.3	33.8	18.1	3.5	-5.2

1. Using the graphing calculator create a scatter plot and regression line of temperatures for Topeka, KS and Barrow, AK. Use the graph below to reproduce what you see on the calculator screen. Be sure to label your axes with variables and scales.



2. Calculate the amplitude using the formula above for each set of data. Compare your answers with the amplitudes from the equation you discovered using the calculator in problem number 1.

Amplitude for Topeka, KS = $\frac{89.3 - 37}{2}$ = 26.15; y = 27.48sin(.46x – 1.69) + 61.70

Amplitude for Barrow, AK = $\frac{42.3 - (-11.8)}{2}$ = 27.05; y = 28.53sin(.58x – 2.70) + 17

3. Compare and discuss the phase shift and amplitude of the two graphs. The discussion should include the similarities and differences of the two graphs.

4. If the following equation represents the pattern of temperature changes in a certain city in the United States, what would the average daily temperature be in June?

$$y = 23.58\sin(.55x - 2.33) + 81.49$$

City	**Jan**	**Feb**	**Mar**	**Apr**	**May**	**Jun**	**Jul**	**Aug**	**Sep**	**Oct**	**Nov**	**Dec**
Las Vegas	**57.3**	**63.3**	**68.8**	**77.5**	**87.8**	**100.3**	**105.9**	**103.2**	**94.7**	**82.1**	**67.4**	**57.5**

Answer: y= 100.3

Assessment

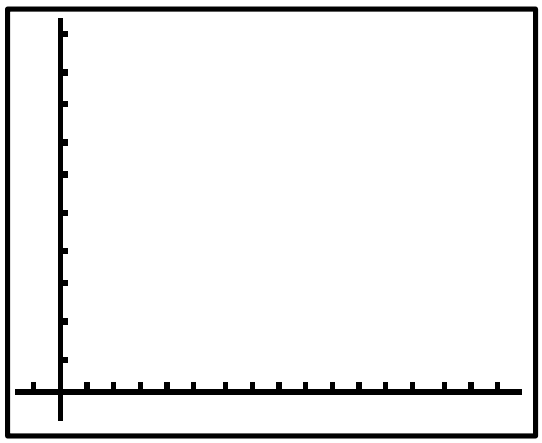
Name: _____ **Date: _____**

1. Give an example of an event that demonstrates periodic behavior.

2. Create a scatter plot using the following data. (Let 10:00 be x=0, 12:00 be x=2 etc.) Display the approximate location of the data points in the window below. Be sure to label your axes with variables and scales. This example represents the depth level of the harbor as a function of tide effects.

Portland, ME

y:



x:

Date/Time (Local Time)	**Observed. depth** (ft.)
06/26/2000 10:00:00 EDT	3.78
06/26/2000 12:00:00 EDT	0.71
06/26/2000 14:00:00 EDT	1.43
06/26/2000 16:00:00 EDT	5.31
06/26/2000 18:00:00 EDT	8.75
06/26/2000 20:00:00 EDT	8.96
06/26/2000 22:00:00 EDT	5.61
06/27/2000 00:00:00 EDT	1.89
06/27/2000 02:00:00 EDT	1.33
06/27/2000 04:00:00 EDT	4.30
06/27/2000 06:00:00 EDT	8.09
06/27/2000 08:00:00 EDT	8.96
06/27/2000 10:00:00 EDT	6.07

3. The appropriate regression model for the data is:
   a. linear
   b. sinusoidal
   c. logistic
   d. quadratic

4. Perform the appropriate regression on the data. Graph the regression equation over the data points above.

5. Write your regression equation below. Round your numbers to hundredths.

_____

6. True/False: The range of the function in #5 is [-1,1].

7. Justify your choice in #6. Explain how you arrived at your answer.

8. Due to the continuous nature of the waxing and waning of tides, the regression equation should allow you to predict, reasonably, the water level of the harbor at a given time. Can a small cargo-boat requiring a depth of at least 3 feet, safely dock in the harbor if the arrival time is 7 am and the time to unload the cargo is 4.5 hours? Explain your answer using mathematics.

Assessment
Key and Scoring Tool

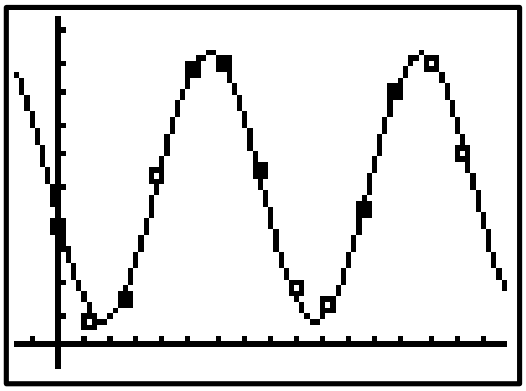
Name: _____ **Date: _____**

1. Give an example of an event that demonstrates periodic behavior.
   (1 point) Teacher discretion, a reasonable example (i.e. cycle of new to full moon, sound waves, electrical currents, sunrise/sunset cycles, etc.)

2. Create a scatter plot using the following data. (Let 10:00 be x=0, 12:00 be x=2 etc.) Display the approximate location of the data points in the window below. Be sure to label your axes with variables and scales. This example represents the depth level of the harbor as a function of tide effects.
   (2 points for similar scatter plot and labels; 1 point for just scatter plot)

Portland, ME

y: depth



x: time

Date/Time (Local Time)	**Observed. depth** (ft.)
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06/27/2000 08:00:00 EDT	8.96
06/27/2000 10:00:00 EDT	6.07

3. The appropriate regression model for the data is: **(1 point)**
   a. linear
   b. sinusoidal
   c. logistic
   d. quadratic

4. Perform the appropriate regression on the data. Graph the regression equation over the data points above. **(1 point for similar graph on above data points)**

5. Write your regression equation below. **(3 points if correct model with correct *a,b,c,d* rounded to hundredths;2 points if correct equation not rounded correctly; 1 point if correct model with incorrect *a,b,c,d*)**

   y = 4.24 sin(0.50x – 2.90) + 5.00

6. True/**False**: The range of the function you got in #5 is [-1,1].
   (1 point)

7. Justify your choice in #6. Explain how you arrived at your answer.
   <u>Option1</u>: Graphically – (2 points) if describe the graph, noting the graph never drops below the x-axis, therefore it is impossible for y≤ 0;
   - (1 point) if only mention of the graph

   <u>Option2</u>: Algebraically - (2 points) if student states the fact that *y = sinx* has a range of [-1,1], therefore *y = a sin (bx-c) +d* has a range of [-a +d, a +d]. In our model a= 4.24, d= 5.0, so that the range of the regression model is [0.76, 9.24]
   - (1 point) if student explains algebraically coherent, but not complete

8. Due to the continuous nature of the waxing and waning of tides, the regression equation should allow you to predict, reasonably, the water level of the harbor at a given time. Can a small cargo-boat requiring a depth of at least 3 feet, safely moor in the harbor if the arrival time is 7am and the time to unload the cargo is 4.5 hours? **Yes**

   (4 points) correct answer and a sound, clear, explicit mathematical explanation, such as
   -if at 10am: x = 0, find x when the time is 7 am; x = 21
   -4.5 hours later, x = 25.5
   -Solve for y when x = 21 → *y = 9.11 ft.* and x = 25.5 → *y = 3.21 ft.*
   -The depth is decreasing but at the time of departure, the depth is still at a safe level.
   (3 points) correct answer, plus some calculations leading up to decision
   (2 points) correct answer and brief explanation about the appearance of the graph, but with no mathematical backing.
   (1 point) just the correct answer